Kenmore-Town of Tonawanda UFSD

We educate, prepare, and inspire all students to achieve their highest potential

Grade 2 Module 6
Parent Handbook

The materials contained within this packet have been taken from the Great Minds curriculum Eureka Math.

Eureka Math™ Tips for Parents

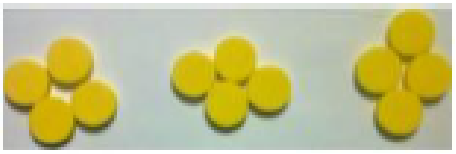
Grade 2
Module 6

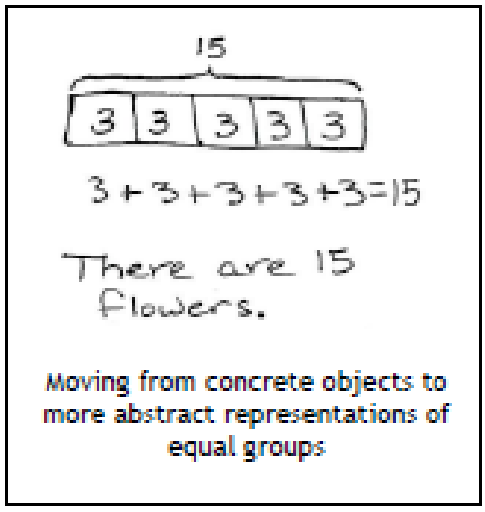
Foundations of Multiplication and Division

Module 6 lays the conceptual foundation for multiplication and division in Grade 3 and the idea that numbers other than 1, 10, and 100 can serve as units.

Students learn to make equal groups, moving from concrete work with objects to more abstract pictorial representations. Finally, they learn about even and odd numbers.

We are learning to make equal groups!





Moving from concrete objects to more abstract representations of equal groups

What Came Before this Module: Students extended their work with addition and subtraction algorithms to numbers up to 1,000. They also worked with word problems with numbers up to 100.

What Comes After this Module: In Module 7, students work on their addition and subtraction skills using units for length, as well as money. They also collect and represent data in various ways, including bar graphs, picture graphs, and line plots.

New Terms in this Module:

Array—arrangement of objects in rows and columns

Columns—the vertical groups in a rectangular array

Even number—a whole number whose last digit is 0, 2, 4, 6, or 8

Odd number—a number that is not even

Repeated addition—e.g., 2 + 2 + 2

Rows—the horizontal groups in a rectangular array

Tessellation—tiling of a plane using one or more geometric shapes with no overlaps and no gaps

Whole number—e.g., 0, 1, 2, 3,...

Familiar Terms:

Addend	Double
Equation	Pair
Rectangle	Skip Counting
Square	Sum
Tape Diagram	Total

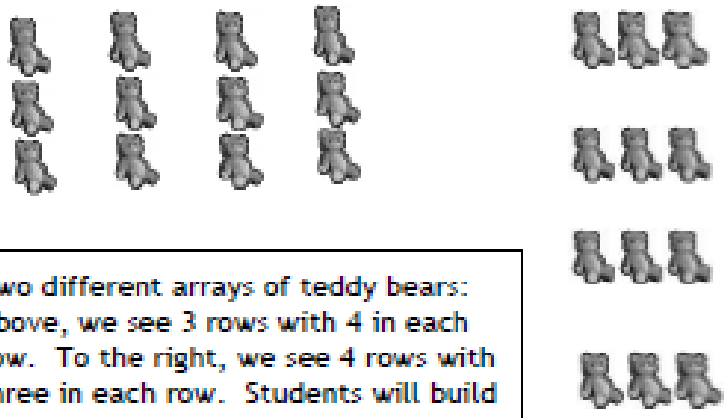
How You Can Help at Home:

- Using any number of small objects, challenge your student to sort them into equal groups.
- Practice skip-counting by 2s. This will help as students work with odd and even numbers in this module.

Key Common Core Standards:

- *Work with equal groups of objects to gain foundations for multiplication.*
  - Determine whether a group of objects (up to 20) has an odd or even number of members; write an equation to express an even number as a sum of two equal addends.
  - Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns.
- *Reason with shapes and their attributes.*
  - Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Prepared by Erin Schweng, Math Coach



Two different arrays of teddy bears: Above, we see 3 rows with 4 in each row. To the right, we see 4 rows with three in each row. Students will build these arrays and understand that both equal a total of 12 and why that is so.

Spotlight on Math Strategies:

Arrays

Students will use this model in Module 6 of *A Story of Units*, as well as throughout their elementary years.

A Story of Units has several key mathematical "models" that will be used throughout a student's elementary years.

An array is an arrangement of objects organized into equal groups in rows and columns. Arrays help make counting easy. Students are reminded in this module that counting by equal groups is more efficient than counting objects one by one. This module focuses on establishing a strong connection between the array and repeated addition (e.g., 3 rows of 4 can be expressed as 4 + 4 + 4 = 12). Beginning in kindergarten, arrays are used as students organize objects into groups to make 10. Now, in Grade 2, we introduce the idea that equal groups can be made of numbers other than 1, 10, or 100.

In Module 6, students build arrays and then use them to write equations showing the repeated addition represented by the array. This lays important groundwork for understanding multiplication as repeated addition in Grade 3. As students progress through their elementary years, arrays will be frequently used to reinforce the relationship between multiplication and division.

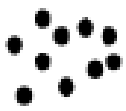
Sample Problem from Module 6:

Redraw the following sets of dots as columns of two or as two equal rows.

(This problem shows how students will be learning about odd and even numbers in Module 6.)

Sample taken from Module 6, Lesson 18

a.



There are _____ dots.

Is ____ an even number? _____

b.

There are _____ dots.

Is ____ an even number? _____

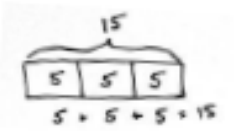
Grade 2 • Module 6

Foundations of Multiplication and Division

OVERVIEW

Grade 2 Module 6 lays the conceptual foundation for multiplication and division in Grade 3 and for the idea that numbers other than 1, 10, and 100 can serve as units.

In Topic A, students begin by making equal groups using concrete materials, learning to manipulate a given number of objects to create equal groups (e.g., given 15 objects, they create 3 groups of 5 or 5 groups of 3), and progress to pictorial representations where they may begin by circling a group of 5 stars, adding 5 more, then adding 5 more. They determine the total and relate their drawings to the corresponding repeated addition equation (pictured below). Students calculate the repeated addition sums by adding on to the previous addends, step-by-step, or by grouping the addends into pairs and adding. By the end of Topic A, students draw abstract tape diagrams to represent the total and to show the number in each group as a new unit (pictured below). Hence, they begin their experience toward understanding that any unit may be counted, e.g., 3 dogs, 3 tens, or even 3 fives. This is the bridge between Grades 2 and 3. Grade 2 focuses on the manipulation of place value units, whereas Grade 3 focuses on the manipulation of numbers 1 through 10 as units.



In Topic B, students organize the equal groups created in Topic A into arrays, wherein either a row or column is seen as the new unit being counted. They use manipulatives to compose up to 5 by 5 arrays one row or one column at a time and express the total via repeated addition equations (**2.OA.4**). For example, students might arrange one column of 5 counters, then another, and then another to compose an array of 3 columns of 5, or 15 counters. As they compose and decompose arrays, students create different number sentences yielding the same total (e.g., 5 + 5 + 5 = 15 and 3 + 3 + 3 + 3 + 3 = 15). They find the total number of objects in each array by counting on from left to right. “Three plus 3 is 6. Six plus 3 is 9. Nine plus 3 is 12." As Topic B progresses, students move to the pictorial level to represent arrays and to distinguish rows from columns by separating equal groups horizontally and vertically (e.g., 3 columns of 5 or 5 rows of 3). Then, they use same-size square tiles, moving them closer together in preparation for composing rectangles in Topic C. Topic B concludes with students using tape diagrams to represent array situations and the RDW process to solve word problems.

In Topic C, students build upon their work with arrays to develop the spatial reasoning skills they need in preparation for Grade 3’s area content. They use same-size squares to tile a rectangle with no gaps or overlaps, and then count to find the total number of squares that make up the rectangle (**2.G.2**). After composing rectangles, students partition, or decompose, rectangles. First, they decompose rectangles made of square tiles. Next, they use scissors to cut apart paper rectangles. Finally, they draw and iterate a square unit. In doing so, students begin to see the row or the column as a composite of multiple squares or as a single entity, or unit, which is, in turn, part of the larger rectangle. Students further develop spatial structuring skills by copying and creating drawings on grid paper. Note that the concept of a square unit begins in Grade 3 and is not assessed in Grade 2. Throughout the topic, students relate repeated addition to

the model. They are encouraged to think flexibly and to consider the many ways to construct or partition a given array. Students are not multiplying or dividing in Grade 2; rather, this topic lays the foundation for the relationship between the two operations. As equal parts can be composed to form a whole, likewise, a whole can be decomposed into equal parts.

Topic D focuses on doubles and even numbers (**2.OA.3**), thus setting the stage for the multiplication table of two in Grade 3. As students progress through the lessons, they learn the following interpretations of even numbers.

1. A number that occurs as we skip-count by twos is even: 2, 4, 6, 8…
2. When objects are paired up with none left unpaired, the number is even.
3. A number that is twice a whole number (doubles) is even.
4. A number whose last digit is 0, 2, 4, 6, or 8 is even.

Armed with an understanding of the term *even,* students learn that any whole number that is not even is called *odd,* and that when 1 is added to or subtracted from an even number, the resulting number is odd.

Initially, students arrange pairs into two rows and realize that an even number is the sum of two equal addends, or a repeated sum of twos. They then write number sentences to express the even number (e.g., 2 rows of 7 can be expressed as 7 + 7 or as 2 + 2 + 2 + 2 + 2 + 2 + 2) (**2.OA.3**). Next, students pair objects to make groups of two with none left over, thus discovering one means of determining whether a group of objects (up to 20) has an even or odd number of members. Finally, students learn that any number up to 20 whose last digit is 0, 2, 4, 6, or 8 is even. After gaining a firm understanding of even numbers, students learn that all other whole numbers are odd. They use the previously learned rules and patterns to identify larger numbers as even or odd and to defend their reasoning. The module concludes with an investigation of what happens when we add two even numbers, two odd numbers, or an odd number with an even number, and the relationship of these pairings to repeated addition (e.g., 3 + 3 is even, but 3 + 3 + 3 is odd).

Terminology

New or Recently Introduced Terms

- Array (an arrangement of objects in rows and columns)
- Columns (the vertical groups in a rectangular array)
- Even number (a whole number whose last digit is 0, 2, 4, 6, or 8)
- Odd number (any number that is not even)
- Repeated addition (e.g., 2 + 2 + 2)
- Rows (the horizontal groups in a rectangular array)
- Tessellation (tiling of a plane using one or more geometric shapes with no overlaps and no gaps)
- Whole number (e.g., 0, 1, 2, 3…)

Familiar Terms and Symbols

- Addends
- Doubles
- Equation
- Number path
- Number sentence
- Pair
- Rectangle
- Skip-counting
- Square
- Sum
- Tape diagram
- Total
- Unit

Suggested Tools and Representations

- Counters
- Number bond
- Number path
- Personal white board
- Rectangular array
- Square tiles

Grade 2 Module 6 Topic A

Formation of Equal Groups

Focus Standard:

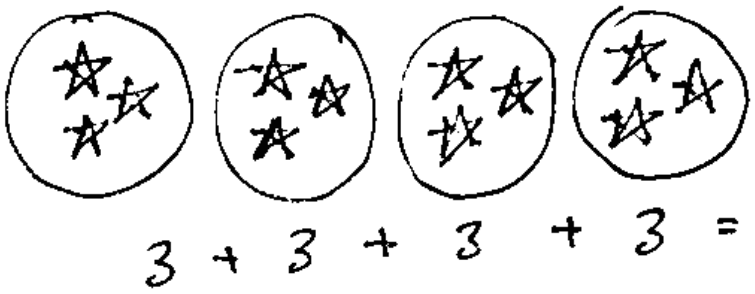
2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Instructional Days Recommended: 4

Topic A begins at the concrete level as students use objects to create equal groups, providing a foundation for the construction of arrays in Topic B. In Lesson 1, for example, students are given 12 counters, such as teddy bears, pebbles, or beans, and they are asked to put them into groups of 3, thereby creating 4 equal groups of 3 objects.

Students then see that they can manipulate the same number of counters to make 3 equal groups of 4 objects. Finally, they are presented with unequal groups and challenged to make them equal.

Lessons 2 and 3 move to the pictorial level, introducing math drawings to represent equal groups. In Lesson 2, students are asked to show groups: “Show me 3 stars, now 3 more. Add 3 more, now 3 more than that.” They then determine the total number of stars and write the corresponding repeated addition number sentence as shown at below (**2.OA.4**).



Lesson 3 extends this understanding as students look for and practice a more efficient way to add by bundling. They calculate repeated addition sums by grouping the addends into pairs and then adding. For example, for 4 groups of 3, the student might say, "I bundled 2 pairs of three to make sixes, so 6 + 6 = 12." If there is an odd number of addends (e.g., 5 groups of 3), students group them into pairs and then add on the remaining quantity, such that (3 + 3) + (3 + 3) = 6 + 6 = 12, and then, 12 + 3 = 15. As students work with equal groups, they begin to see that they are adding units of 3.

This concept transitions into Lesson 4, where students understand that numbers other than 1, 10, and 100 can serve as units. At a more abstract level than Lesson 3, students represent the total of a given number of units with tape diagrams or using repeated addition (e.g., 2 + 2 + 2 + 2 = 8). This concept serves as a bridge to Topic B, wherein either a row or column of an array can be seen as the unit being counted—the foundation for building rectangular arrays (**2.OA.4**).

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 1

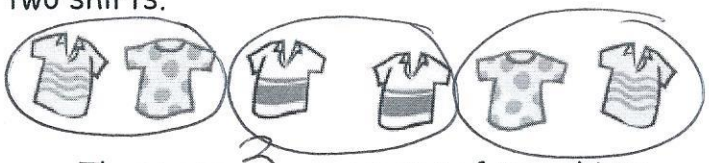
Objective: Use manipulatives to create equal groups.

Homework Key

1. 3 groups of 2 shirts circled; 3
2. 4 groups of 3 pants circled; 4
3. 3 groups of 4 wheels drawn; 4
4. 4 groups of 3 wheels drawn; 3
5. 4 groups of 2 apples drawn; 2, 8
6. 3 groups of 5 oranges drawn; 5, 15

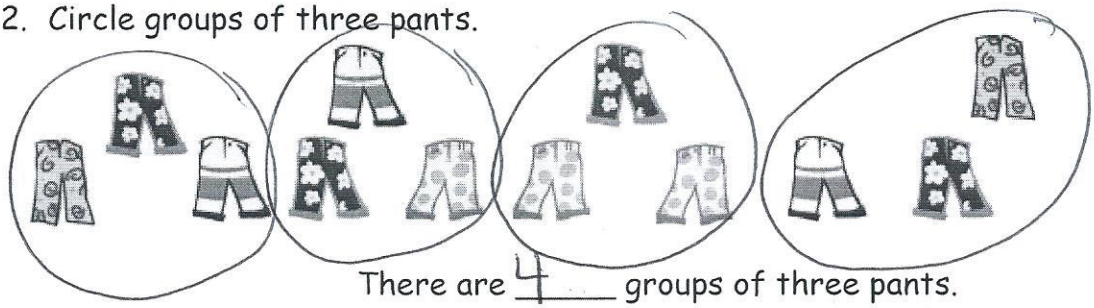
Homework Sample

1. Circle groups of two shirts.



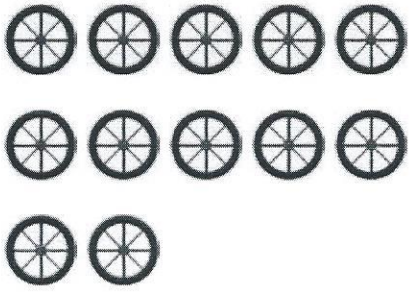
There are 3 groups of two shirts.

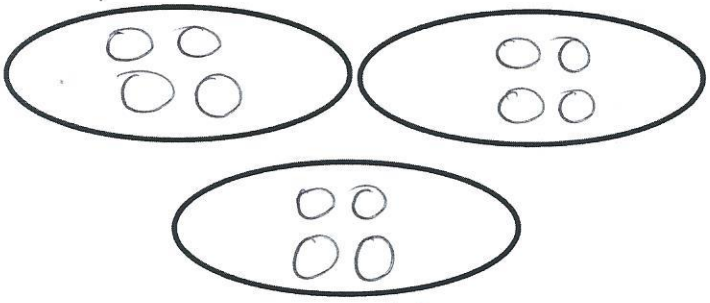
2. Circle groups of three pants.



There are 4 groups of three pants.

3. Redraw the 12 wheels into 3 equal groups.





3 groups of 4 wheels

Lesson 2 - 3

Objective: Use math drawings to represent equal groups, and relate to repeated addition.

Homework Key (2)

1. a. 3, 3, 3, 9; 3, 9
   b. 2, 2, 2, 2, 8; 2, 8
2. 1 group of 3 cars drawn; 3, 3, 3, 3, 3, 15; 3, 15
3. 1 group of 4 triangles drawn; 4, 4, 4, 4, 16; 4, 16
4. 2 groups of 4 hearts drawn; 4, 4, 4, 4, 4, 20; 5, 20
5. 4 groups of 3 circles drawn; 3 + 3 + 3 + 3 = 12

Homework Sample

1. Write a repeated addition equation to show the number of objects in each group. Then, find the total.

a. 

3 + 3 + 3 = 9

3 groups of 3 = 9

b. 

2 + 2 + 2 + 2 = 8

4 groups of 2 = 8

Lesson 3

Homework Key

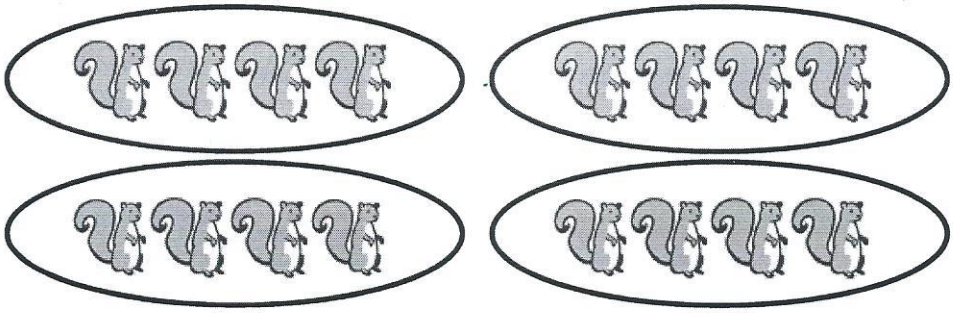
1. a. 4, 4, 4, 4, 16
   8, 8, 16
   4, 8
   b. 2, 2, 2, 2, 8
   4, 4, 8
   2, 4
   c. 5, 5, 5, 5, 20
   10, 10, 20
   5, 10

2. a. 3, 3, 3, 3, 3, 15
   6, 6, 15
   12, 15
   b. 2, 2, 2, 2, 2, 10
   4, 4, 10
   8, 10

Homework Sample

1. Write a repeated addition equation to match the picture. Then, group the addends into pairs to show a more efficient way to add.

a.



4 + 4 + 4 + 4 = 16

(8) + (8) = 16

4 groups of 4 = 2 groups of 8

Lesson 4

Objective: Represent equal groups with tape diagrams, and relate to repeated addition.

Homework Key

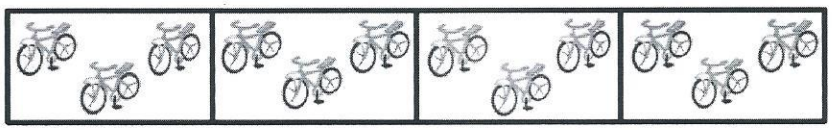
1. a. 3, 3, 3, 3, 12; 12
   b. 5, 5, 5, 5, 5, 25; 5, 25
   c. 4, 4, 4, 4, 16; 4, 16
   d. 2, 2, 2, 2, 2, 2, 12; 6, 2, 12

2. a. 20; tape diagram drawn to show 4 fives
   b. 20; tape diagram drawn to show 5 fours
   c. 8; tape diagram drawn to show 4 twos
   d. 15; tape diagram drawn to show 5 threes
   e. 12; tape diagram drawn to show 3 fours

Homework Sample

1. Write a repeated addition equation to find the total of each tape diagram.

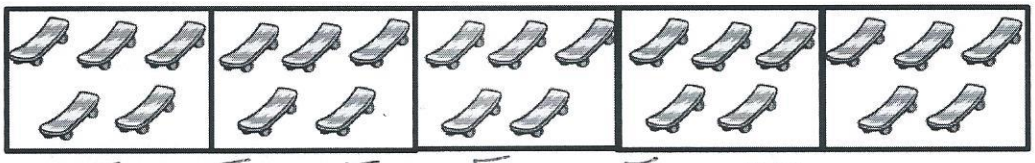
a.



3 + 3 + 3 + 3 = 12

4 groups of 3 = 12

b.



5 + 5 + 5 + 5 + 5 = 25

5 groups of 5 = 25

Grade 2 Module 6 Topic B

Arrays and Equal Groups

Focus Standards:

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

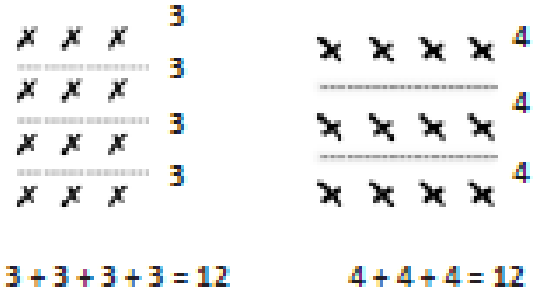
Instructional Days Recommended: 5

Topic B focuses on spatial relationships and structuring as students organize equal groups (from Topic A) into rectangular arrays. They build small arrays (up to 5 by 5) and use repeated addition of the number in each row or column (i.e., group) to find the total.

In Lesson 5, students compose arrays either one row or one column at a time and count to find the total using the scattered sets from Topic A. For example, they might arrange one row of 3 counters, followed by three more identical rows, to compose a 4 by 3 array of 12 counters. Then, students use the same equal groups to create an array, column by column (shown below). They count to find the total, noticing that each row and each column contain the same number of units. Thus, for 4 rows of 3 or 4 columns of 3, a student might observe, “There are 4 equal groups of 3.” This is foundational to the spatial structuring students need to discern a row or column as a single entity, or unit, when working with tiled arrays without gaps and overlaps in Topic C.

In Lesson 6, students decompose one array by both rows and columns. Thus, an array of 4 rows of 3 teddy bears can be pulled apart to show either 4 rows of 3 or 3 columns of 4. Also, students see that when another row or column is added or removed, so is another group, or unit. As Lesson 6 progresses, students move the objects of the arrays closer together so the gaps are smaller, forcing them to discern the rows and columns without the visual aid of spacing. For example, when decomposing a 4 by 3 array, students see the rows as equal groups of 3. After identifying the number in each row, or group, students realize that they can write a repeated addition sentence to find the total number of objects in the array: 3 + 3 + 3 + 3 = 12. It may be noted that since there are 4 rows, the equation will have 4 addends, or 4 threes. Students add from left to right and write the sum, such that 3 plus 3 equals 6, 6 plus 3 equals 9, and 9 plus 3 equals 12.

In Lesson 7, students move to the pictorial as they use math drawings to represent arrays and relate the drawings to repeated addition. For example, students are asked to draw an array with 4 rows of 3 or 3 rows of 4 on their personal white board, and then use their marker to draw horizontal lines to see the rows within the array (shown below). When counting rows containing 3 or 4 objects, students apply repeated addition strategies once again, adding from left to right to find the sum (e.g., 4 + 4 + 4 = 12, such that 4 plus 4 equals 8 and 8 plus 4 equals 12). Additionally, when representing arrays with rows of 2 or 5, students may add to find the total and naturally point out a connection to skip-counting by twos or fives (**2.NBT.2**); the focus, however, is on establishing a strong connection between the array and repeated addition.



In Lesson 8, students work with square tiles to create arrays with gaps, composing the arrays from parts to whole, either one row or one column at a time. Seeing arrays as composed of individual, separated tiles provides the foundation for Topic C, where students work with square tiles without gaps. As usual, students relate the arrays to repeated addition.

In Lesson 9, students apply the work of Topic B to word problems involving repeated addition (shown below), interpreting array situations as either rows or columns and using the RDW process, e.g., “Mrs. Levy moves desks into 3 columns of 4 desks. How many desks does she move?” In addition to drawing objects, students may also represent the situation with more abstract tape diagrams, just as they did in the final lesson of Topic A.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 5

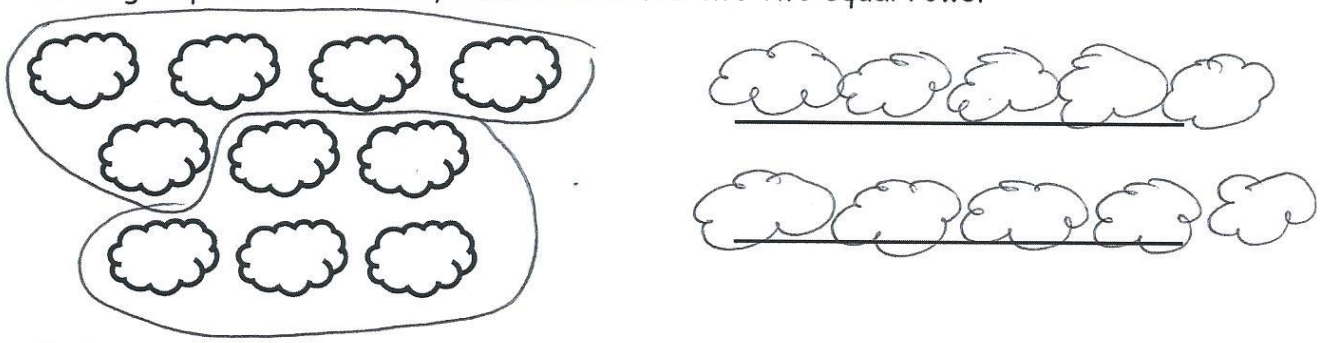
Objective: Compose arrays from rows and columns, and count to find the total using objects.

Homework Key

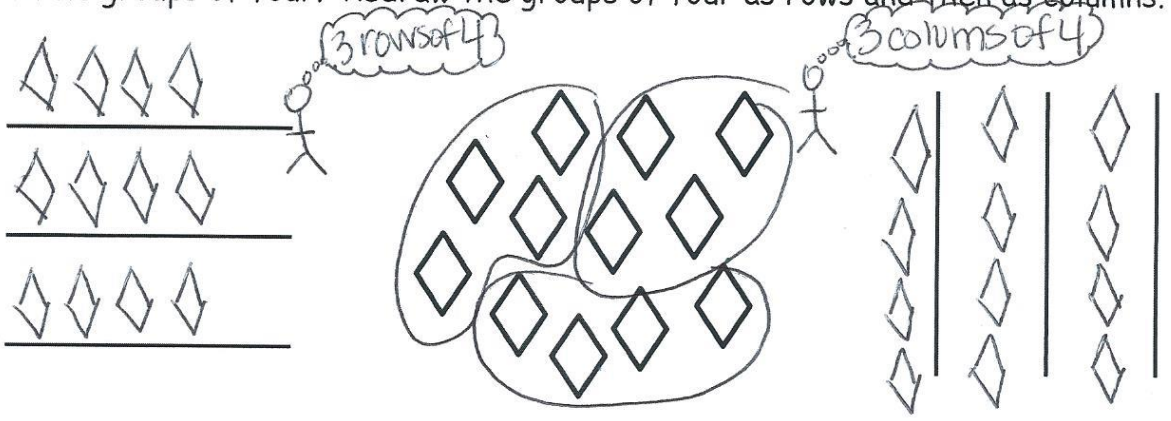
1. 2 groups of 5 clouds circled; 2 rows of 5 clouds drawn
2. 3 groups of 4 diamonds circled; 3 rows and 3 columns of 4 diamonds each drawn
3. 4 groups of 4 circles circled; 4 rows and columns of 4 circles each drawn
4. a. 5 rows and 3 columns circled
   b. 4 rows and 3 columns circled
5. 5 columns of 3 rows of smiley faces drawn; 4 columns of 3 rows of triangles drawn
6. Answers will vary.
7. Answers will vary.

Homework Samples

1. Circle groups of five. Then, draw the clouds into two equal rows.



2. Circle groups of four. Redraw the groups of four as rows and then as columns.



Lesson 6

Objective: Decompose arrays from rows and columns, and relate to repeated addition.

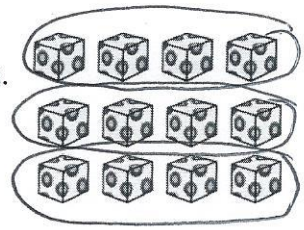
Homework Key

1. a. 3 rows of 4 circled; 4, 12; 4, 4, 4, 12
   b. 4 columns of 3 circled; 3, 12; 3, 3, 3, 3, 12
   c. 5 rows of 3 circled; 3, 15; 3, 3, 3, 3, 3, 15
   d. 3 columns of 5 circled; 5, 15; 5, 5, 5, 15
2. a. 3, 5, 15
   b. 5, 3, 15
   c. 5, 5, 5, 15
   d. 1 more row of 5 smiley faces added; 20
   e. 1 more column of 4 smiley faces added; 24
3. a. 3, 3, 3, 3, 12
   b. 4, 3, 12
   c. 3, 4, 12
   d. 1 row of 3 crossed out; 9
   e. 1 column of 3 crossed out; 6

Homework Sample

1. Complete each missing part describing each array.

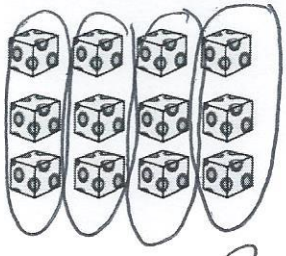
Circle rows.

a. 

3 rows of 4 = 12

4 + 4 + 4 = 12

Circle columns.

b. 

4 columns of 3 = 12

3 + 3 + 3 + 3 = 12

Lesson 7

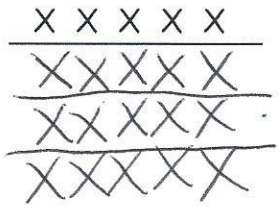
Objective: Represent arrays and distinguish rows and columns using math drawings.

Homework Key

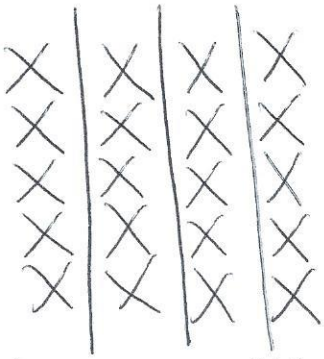
1. a. Array of 4 rows of 5 Xs completed
   b. Array with 4 columns of 5 Xs drawn; 5, 5, 5, 5, 20; 20; 20
2. a. Array of 3 columns of 4 Xs drawn
   b. Array of 3 rows of 4 Xs drawn; 4, 4, 4, 12; 12; 12
3. Array with 3 rows of 3 Xs drawn; 3, 3, 3, 9; 9
4. Array with 5 rows of 3 Xs drawn; 3 + 3 + 3 + 3 + 3 = 15
5. Array with 2 columns of 5 Xs drawn; 2 + 2 + 2 + 2 + 2 = 10 or 5 + 5 = 10

Homework Sample

1. a. One row of an array is drawn below. Complete the array with X's to make 4 rows of 5. Draw horizontal lines to separate the rows.



b. Draw an array with X's that has 4 columns of 5. Draw vertical lines to separate the columns. Fill in the blanks.



5 + 5 + 5 + 5 = 20

4 rows of 5 = 20

4 columns of 5 = 20

Lesson 8

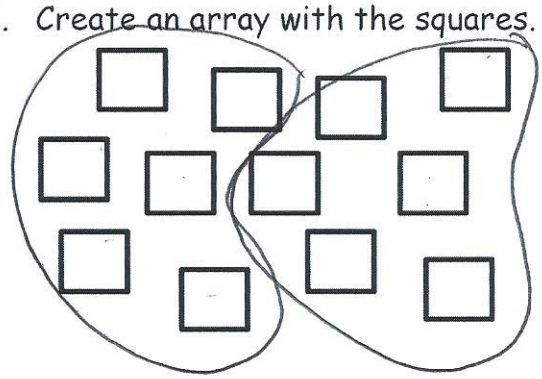
Objective: Create arrays using square tiles with gaps.

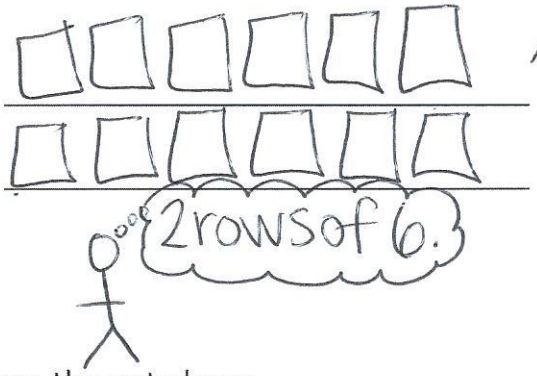
Homework Key

1. Array with 3 rows of 4 squares drawn
2. Array with 4 columns of 3 squares drawn
3. a. 5
   b. 5, 5, 5, 15
   c. 3
   d. 3, 3, 3, 3, 3, 15
4. a. 2
   b. 5
   c. 5, 5, 10
   d. 5, 5, 2, 10
5. a. Array with 5 columns of 3 squares drawn
   b. 3 + 3 + 3 + 3 + 3 = 15 or 5 + 5 + 5 = 15
6. a. Array with 4 columns of 5 squares drawn
   b. 4 + 4 + 4 + 4 + 4 = 20 or 5 + 5 + 5 + 5 = 20
   c. Tape diagram drawn

Homework Sample

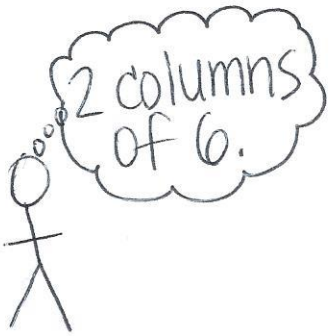
1. Create an array with the squares.

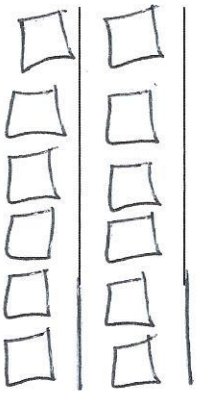




Answers may vary.

2. Create an array with the squares from the set above.





Answers may vary.

Lesson 9

Objective: Solve word problems involving addition of equal groups in rows and columns.

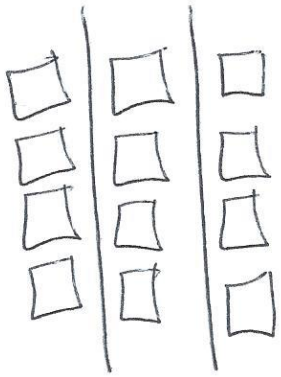
Homework Key

1. Array with 3 columns of 4 drawn, 4 + 4 + 4 = 12 or 3 + 3 + 3 + 3 = 12
2. Array with 5 rows of 5 drawn, 5 + 5 + 5 + 5 + 5 = 25
3. Array with 5 rows of 4 drawn, 4 + 4 + 4 + 4 + 4 = 20 or 5 + 5 + 5 + 5 = 20
4. Array with 4 rows of 4 drawn, 4 + 4 + 4 + 4 = 16
5. Tape diagram drawn to show 5 fours, 4 + 4 + 4 + 4 + 4 = 20
6. Tape diagram drawn to show 5 threes, 3 + 3 + 3 + 3 + 3 = 15
7. Tape diagram and array drawn to show 3 threes, 3 + 3 + 3 = 9

Homework Sample

Draw an array for each word problem. Write a repeated addition equation to match each array.

1. Melody stacked her blocks in 3 columns of 4. How many blocks did Melody stack in all?



4+4+4=12

Melody stacked 12 blocks.

Grade 2 Module 6 Topic C

Rectangular Arrays as a Foundation for Multiplication and Division

Focus Standards:

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Instructional Days Recommended: 7

Topic C naturally follows Topic B, where students composed and manipulated the rows and columns of an array. Topic C is designed to deepen students' understanding of spatial structuring as they build and partition rectangles with rows and columns of same-size squares.

In Lessons 10 and 11, students compose a rectangle by making tile arrays with no gaps or overlaps. They use their prior knowledge of making equal groups and the spatial relationship between rows and columns to construct rectangular arrays. In Lesson 10, given a number of tiles (up to 25), students are asked to create rectangular arrays that show equal rows or columns (up to 5 by 5). In Lesson 11, students build upon this understanding, manipulating a set of 12 square tiles to compose various rectangles (e.g., 1 column of 12, 2 rows of 6, or 3 rows of 4). As students share their rectangles, they are encouraged to ask themselves, "How can I construct this differently?" They use repeated addition to find the total number

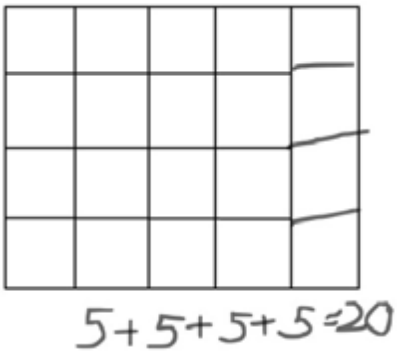
of squares, alternating flexibly between the number in each row and the number in each column as the unit.

Lesson 12 introduces the added complexity of composing a rectangle by using math drawings. Once students have arranged square tiles into a specified rectangular array without gaps or overlaps, they trace to construct the same rectangle by iterating the square unit, much as they iterated a length unit in Module 2 to create a centimeter ruler. Next, students use the spatial reasoning developed up to this point in the module to draw the same rectangle without tracing, using their understanding of equal columns and equal rows.

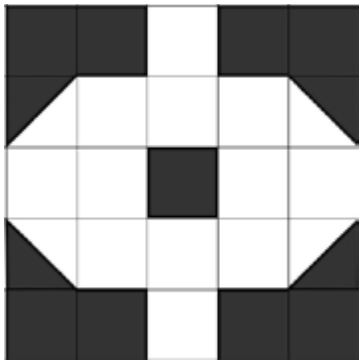
After students compose rectangles, they decompose, or partition, them using tiles in Lesson 13. For example, when working with an array of 5 rows of 3 (and a total of 15), they see that if they remove a row of 3, they have 4 rows of 3 (and a total of 12). Alternately, they see that instead of 3 columns of 5, they have 3 columns of 4.

In Lesson 14, students are encouraged to think flexibly as they use paper models to further develop their ability to visualize arrays. Given three 2 by 4 rectangles, students cut the first rectangle into 2 rows of 4 squares and the second rectangle into 4 columns of 2 squares. They use these models to answer questions and to analyze similarities and differences. Next, they cut each row or column into individual square units. As a result, they see that just as a rectangle is composed of equal rows or columns, each row or column is composed of squares, or iterated units. Students now have 16 same-size squares and can create different rectangular arrays with them (e.g., 1 by 16, 2 by 8, and 4 by 4). Finally, students cut out the squares from the third rectangle and create rectangular arrays using 24 square units.

Lesson 15 moves toward more abstract reasoning as students use math drawings to partition rectangles. With colored pencils and grid paper, students shade in rows or columns and relate them to the repeated addition equation (e.g., 5 rows of 3 squares = 3 + 3 + 3 + 3 + 3, or 5 threes). Then, given a rectangle with one row or one column missing, students draw in the remaining squares to complete the array (shown below) and find the total by relating their completed array to repeated addition.



In Lesson 16, students practice spatial structuring skills by working with grids and diagrams. They copy designs using same-size squares and triangles (half of the squares) as manipulatives. Students create their copies on paper with grid squares of the same size as the manipulative square (shown below). To successfully create these, students must pay careful attention to which grid square to color and how many spaces to leave. Finally, students use grid paper to design a tessellation using a core square composed of a 3 by 3 array of same-size squares. They create designs by coloring the 9 squares and then iterating that core unit. This provides students with the opportunity to sharpen their spatial structuring skills because they must count rows and columns to successfully create a quilt of their designs.



**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 10 - 11

Objective: Use square tiles to compose a rectangle, and relate to the array model.

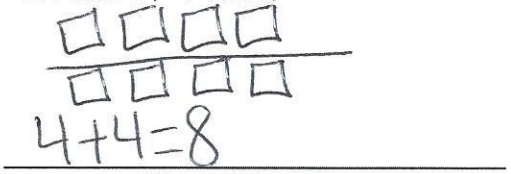
Homework Key (10)

1. a. 4 + 4 = 8
   b. 4 + 4 = 8 or 2 + 2 + 2 + 2 = 8
2. a. 2 + 2 + 2 = 6
   b. 2 + 2 + 2 = 6 or 3 + 3 = 6
3. a. Answers will vary.
   b. Answers will vary.
4. a. Rectangle
   b. Shape is appropriately drawn.
   c. Square
   d. Answers will vary.

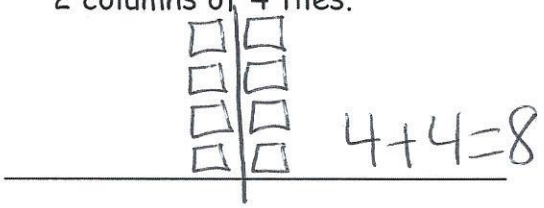
Homework Sample

Cut out the square tiles below, and construct the following arrays with no gaps or overlaps. On the line, write a repeated addition equation to match each construction on the line.

1. a. Construct a rectangle with 2 rows of 4 tiles.



b. Construct a rectangle with 2 columns of 4 tiles.



Lesson 11

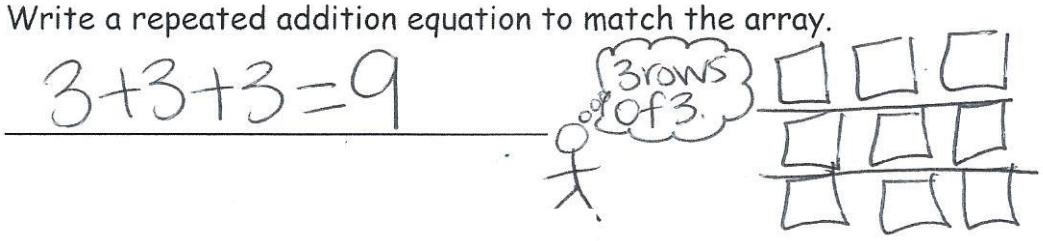
Homework Key

1. a. Array appropriately constructed
   b. Answers will vary.
2. a. Array appropriately constructed
   b. Answers will vary.
   c. Different array appropriately constructed
   d. Answers will vary.
3. a. Array appropriately constructed
   b. Answers will vary.
   c. Different array appropriately constructed
   d. Answers will vary.
4. a. 7, 14
   b. 7, 2

Homework Sample

1. a. Construct an array with 9 square tiles.

   b. Write a repeated addition equation to match the array.



Lesson 12

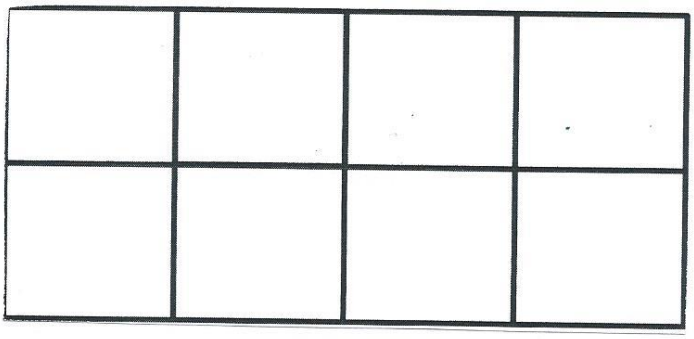
Objective: Use math drawings to compose a rectangle with square tiles.

Homework Key

1. Array appropriately drawn; 8; 4, 4, 8
2. Array appropriately drawn; 15; 5, 5, 5, 15
3. a. Array appropriately completed
   b. Array appropriately completed
   c. Array appropriately completed

Homework Sample

1. Cut out and trace the square tile to draw an array with 2 rows of 4.



Cut out and trace.

2 rows of 4 = 8

4 + 4 = 8

Lesson 13

Objective: Use square tiles to decompose a rectangle.

Homework Key

1. Number bond shown; 2 + 2 = 4, 2 + 2 + 2 = 6
2. Number bond shown; 3 + 3 = 6, 3 + 3 = 6
3. a. 3, 3, 9
   b. 6
   c. 4
4. a. Answers will vary.
   b. Answers will vary.
   c. Answers will vary.

Homework Sample

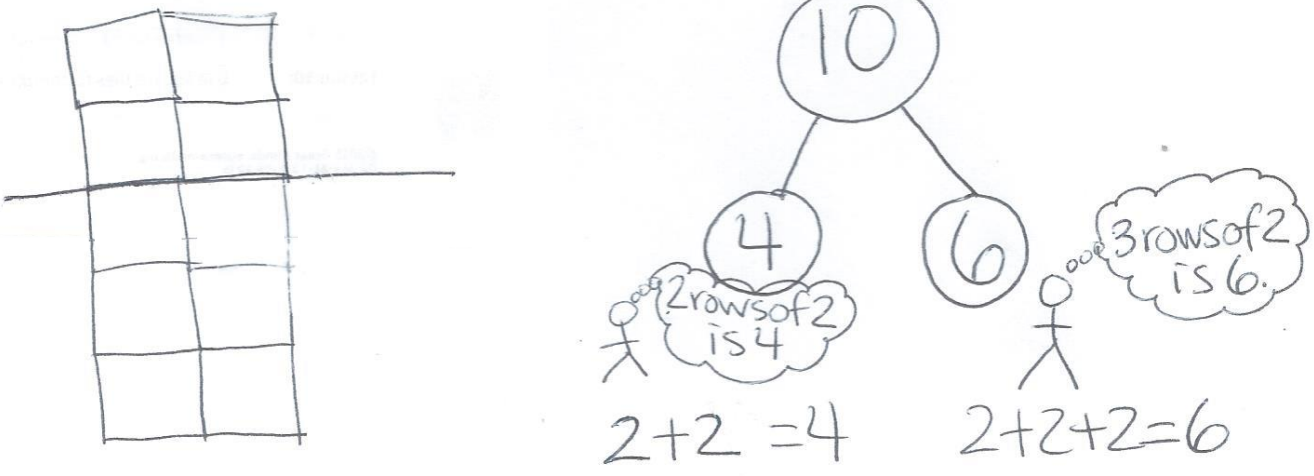
Cut out and use your square tiles to complete the steps for each problem.

Problem 1

Step 1: Construct a rectangle with 5 rows of 2.

Step 2: Separate 2 rows of 2.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of your number bond.



Lesson 14

Objective: Use scissors to partition a rectangle into same-size squares, and compose arrays with the squares.

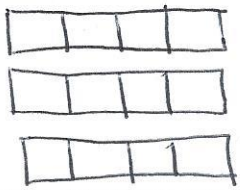
Homework Key

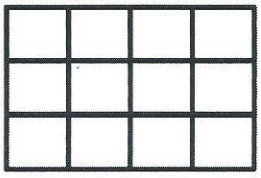
1. a. Picture drawn appropriately; 4
   b. Picture drawn appropriately; 3
2. Answers will vary.
3. a. Picture drawn appropriately; 5
   b. Picture drawn appropriately; 4
4. Answers will vary.

Homework Sample

1. Imagine that you have just cut this rectangle into rows.

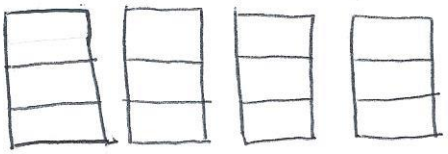
   a. What do you see? Draw a picture.

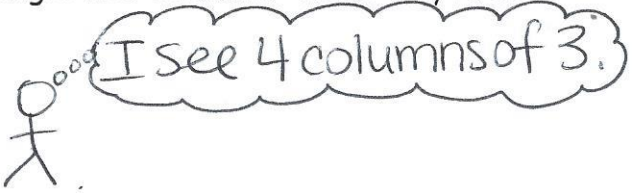




   How many squares are in each row? 4

   b. Imagine that you have just cut this rectangle into columns. What do you see? Draw a picture.





   How many squares are in each column? 3

Lesson 15

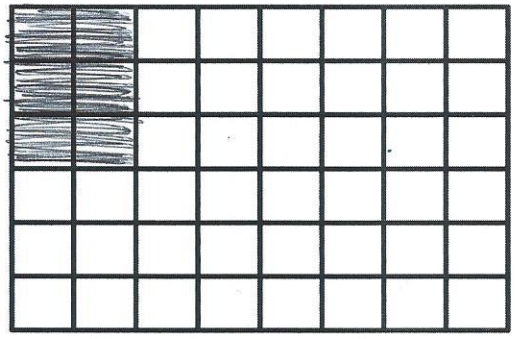
Objective: Use math drawings to partition a rectangle with square tiles, and relate to repeated addition.

Homework Key

1. Array appropriately shaded; 2 + 2 + 2 = 6 or 3 + 3 = 6
2. Array appropriately shaded; 4 + 4 = 8 or 2 + 2 + 2 + 2 = 8
3. Array appropriately shaded; 5 + 5 + 5 + 5 = 20 or 4 + 4 + 4 + 4 + 4 = 20
4. Column appropriately drawn; 2 + 2 + 2 + 2 = 8 or 4 + 4 = 8
5. Row and column appropriately drawn; 4 + 4 + 4 = 12 or 3 + 3 + 3 + 3 = 12
6. Row and columns appropriately drawn; 4 + 4 + 4 + 4 + 4 = 20 or 5 + 5 + 5 + 5 = 20

Homework Sample

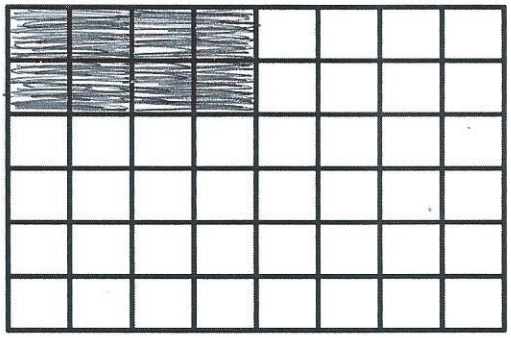
1. Shade in an array with 3 rows of 2.



Write a repeated addition equation for the array.

2+2+2=6

2. Shade in an array with 2 rows of 4.



Write a repeated addition equation for the array.

4+4=8

Lesson 16

Objective: Use grid paper to create designs to develop spatial structuring.

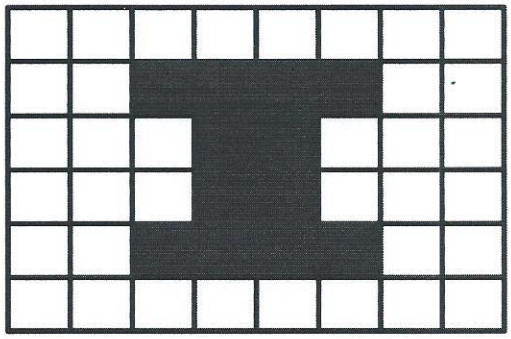
Homework Key

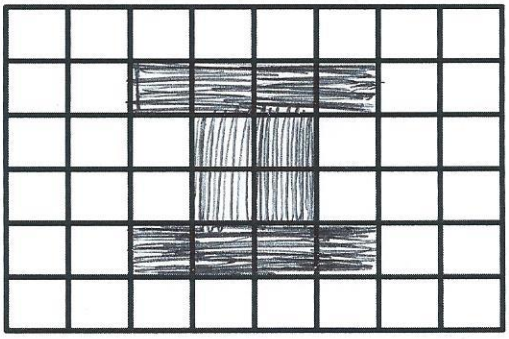
1. a. Design appropriately copied and shaded on the empty grid
   b. Design appropriately copied and shaded on the empty grid
   c. Design appropriately copied and shaded on the empty grid
2. Two different designs appropriately created
3. Design appropriately created in the bold square section; design repeated throughout to create a tessellation

Homework Sample

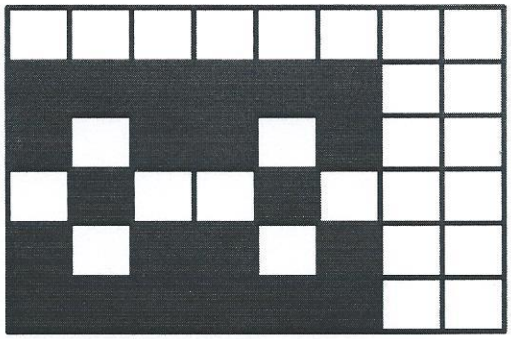
1. Shade to create a copy of the design on the empty grid.

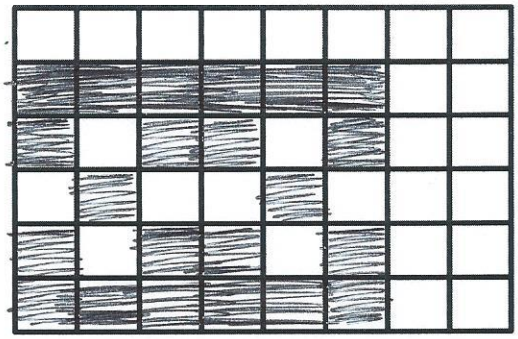
a.





b.





Grade 2 Module 6 Topic D

The Meaning of Even and Odd Numbers

Focus Standard:

2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Instructional Days Recommended: 4

In Topic D, students explore the meaning of even and odd numbers, learning various interpretations and relating these interpretations to addition. Lesson 17 introduces even numbers via doubles. In other words, when any number from 1 to 10 is doubled, the resulting number is even, and any even number can be written as a doubles fact. Students arrange doubles into rectangular arrays (e.g., 2 rows of 7, or 2 sevens) and write an equation to show the total as a sum of two equal addends (e.g., 7 + 7 = 14). They discover that doubles facts yield even numbers even when the number being doubled is not even.

In Lesson 18, students pair up to 20 objects and see that, when objects are paired with none remaining, the number is even (**2.OA.3**). They see that objects arranged in columns of two also create two equal groups. For example, a 2 by 7 array may be seen as 7 columns of 2 or 2 rows of 7. Students also see that even numbers occur when we count by twos (e.g., 1 two, 2 twos, …7 twos, or 2, 4, 6, …14). They count by twos up to 20 and then back down. When they reach zero, the question is posed: “Does this mean zero is even? Can I write 0 as a doubles fact?” As a result, students see that 0 is even. This practice lays the groundwork for the multiplication table of two in Grade 3.

By Lesson 19, students have a keen understanding of how to determine whether or not a number is even. Now, they learn a faster way to identify even numbers by looking for 0, 2, 4, 6, or 8 in the ones place. First, students use square tiles to build an array made up of columns of 2 and relate it to even numbers on a number path up to 20. As multiples of two are circled on the number path, students observe that the ones digits are 0, 2, 4, 6, and 8. Now, equipped with this interpretation, as well as the previously learned interpretations of even numbers, students are ready to name all other whole numbers as odd. They learn that odd numbers can be identified in contrast to even (i.e., if a number is not even, then it is odd). Next, they learn that when one is added to or subtracted from any even number, the resulting number is odd. Finally, students apply all of the interpretations they have learned to discern whether or not numbers larger than 20 are even. For example, a student might reason that 41 is odd because it is an even number plus one or because it does not end in 0, 2, 4, 6, or 8.

Topic D culminates with students using arrays to investigate even and odd numbers. Students build arrays, as they did in Lesson 17, using even numbers, and they see concretely that when a number is even, it can be decomposed into two equal parts or groups of two. They then either remove or add on 1 square unit to make an odd number. This enables students to solidify the understanding that an odd number is either one more or one less than an even number, and that it cannot be decomposed into two equal groups or groups of two, although it may be decomposed into more than two groups (e.g., a 3 by 3 array). Students add even numbers to other even numbers, odd numbers to other odd numbers, and even numbers to odd numbers to see what happens to the sum in each case. They discover that the sum of two even numbers is even, and the sum of two odd numbers is even. They also discover that the sum of an odd number and an even number is odd. Through these explorations, students build an intuitive understanding of prime, composite, and square numbers, which is foundational for later grade levels.

Lesson 17

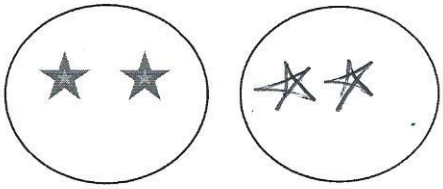
Objective: Relate doubles to even numbers, and write number sentences to express the sums.

Homework Key

1. a. 2 stars drawn; 2; 2, 2, 4
   b. 4 stars drawn; 4; 4, 4, 8
   c. 1 star drawn; 1; 1, 1, 2
   d. 3 stars drawn; 3; 3, 3, 6
   e. 5 stars drawn; 5; 5, 5, 10
2. a. 12; 6, 6, 12; 12
   b. Array appropriately drawn; 14; 7, 7, 14; 14
   c. Array appropriately drawn; 2, 8, 16; 8, 16; 16
   d. Array appropriately drawn; 18; 9, 9, 18; 18
   e. Array appropriately drawn; 2, 10, 20; 10, 20; 20
3. 4, 6, 2, 8, 10
   12, 14, 16, 18, 20
   Even
   Answers will vary.

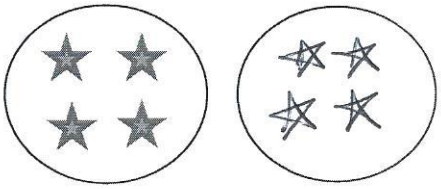
Homework Sample

1. Draw to double the group you see. Complete the sentences, and write an addition equation.

a. 

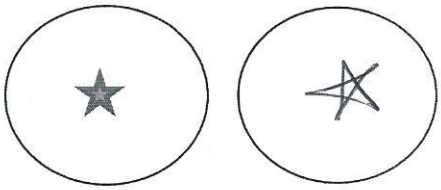
There are 2 stars in each group.

2 + 2 = 4

b. 

There are 4 stars in each group.

4 + 4 = 8

c. 

There is 1 star in each group.

1 + 1 = 2

Lesson 18

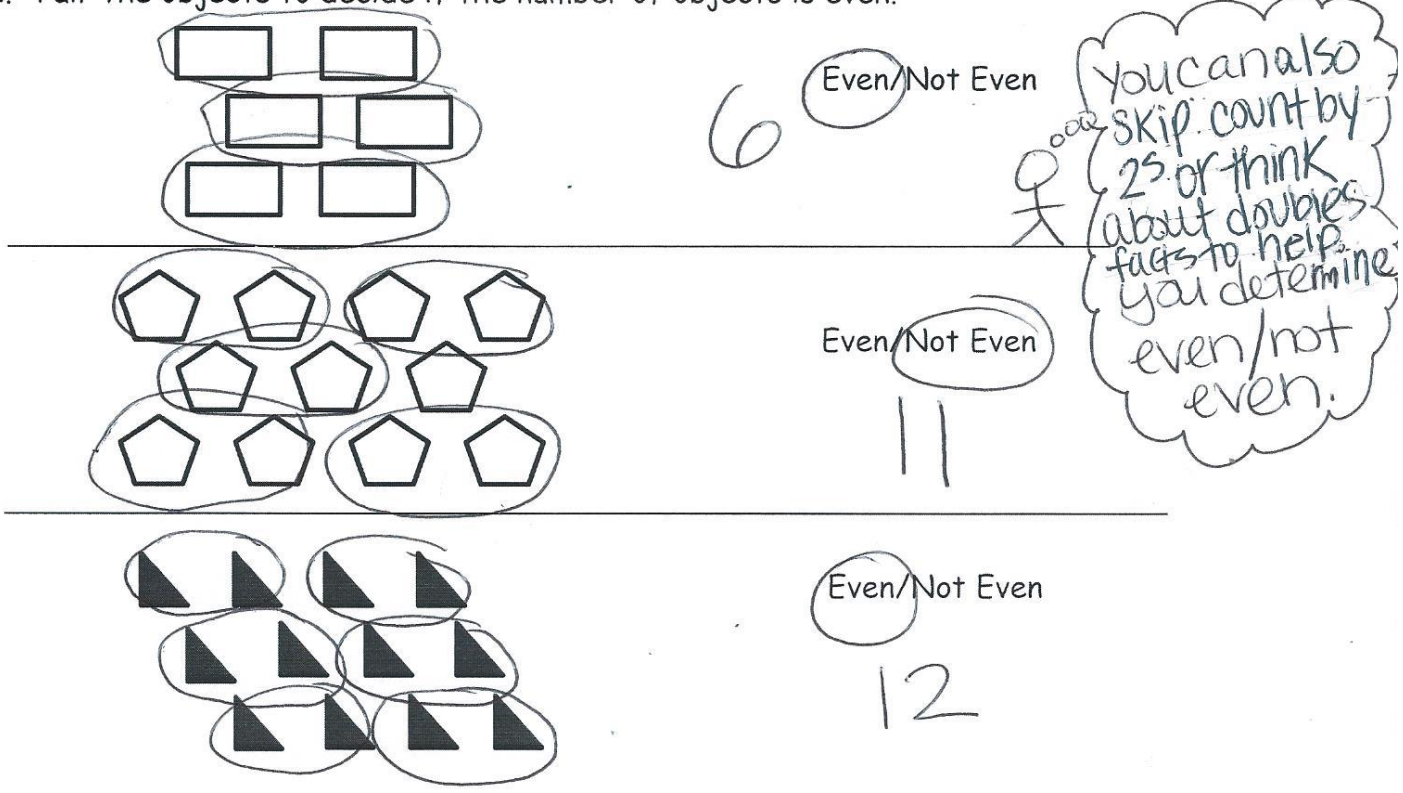
Objective: Pair objects and skip-count to relate to even numbers.

Homework Key

1. Pairs appropriately circled; even
   Pairs appropriately circled; not even
   Pairs appropriately circled; even
2. Pattern appropriately drawn
3. 18, 16, 14, 12, 10, 8, 6, 4, 2
4. Array appropriately circled
5. Array appropriately boxed
6. Stars appropriately redrawn; 12; 12; yes
7. a. 8; 0
   b. 2, 4, 6, 8, 10, 12, 14, 16
   c. True

Homework Sample

1. Pair the objects to decide if the number of objects is even.



Lesson 19

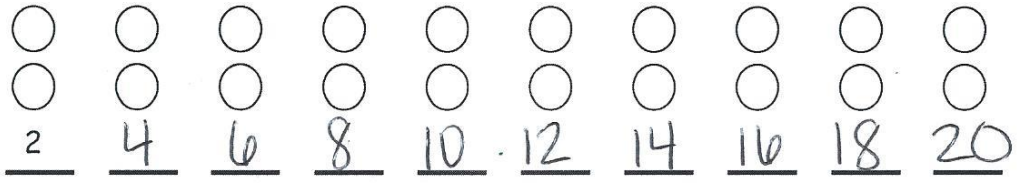
Objective: Investigate the pattern of even numbers: 0, 2, 4, 6, and 8 in the ones place, and relate to odd numbers.

Homework Key

1. 4, 6, 8, 10, 12, 14, 16, 18, 20
2. a. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
   b. Answers will vary.
3. 22, 24; 28; 32; 36; 42, 44
4. 1, 3, 5, 7, 9, 11, 13
5. a. Answer provided
   b. Odd, even
   c. Even, odd
   d. Even, odd
   e. Even, odd
   f. Even, odd
6. a. Odd, explanations will vary.
   b. Even, explanations will vary.

Homework Sample

1. Skip-count the columns in the array. The first one has been done for you.



2. a. Solve.

1 + 1 = 2
2 + 2 = 4
3 + 3 = 6
4 + 4 = 8
5 + 5 = 10
6 + 6 = 12
7 + 7 = 14
8 + 8 = 16
9 + 9 = 18
10 + 10 = 20

b. How is the array in Problem 1 related to the answers in Problem 2(a)?

The doubles facts answers were counting up by 2s just like Problem 1. Example: 7+7=14, you could make 2 rows of 7 or 7 columns of 2 and everyone would have a partner. Therefore, all doubles facts are even and count up by 2s.

Lesson 20

Objective: Use rectangular arrays to investigate odd and even numbers.

Homework Key

1. a. Array appropriately drawn with 12 stars; even

   Picture appropriately drawn with 11 stars; odd

   b. Array appropriately drawn with 14 stars; even

   Picture appropriately drawn with 15 stars; odd

   c. Array appropriately drawn with 14 stars; even

   Picture appropriately drawn with 13 stars; odd

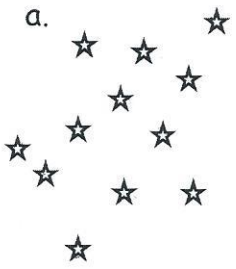
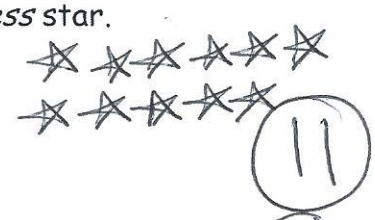
2. a. 12; E; E; E
   b. 21; E; O; O
   c. 24; O; O; E
   d. 25; O; E; O
   e. 15; O; E; O
   f. 20; O; O; E
   g. 21; O; E; O
   h. 18; O; O; E

3. Answers will vary.

4. Terms and sums will vary.
   a. Answers are even.
   b. Answers are odd.
   c. Answers are even.

Homework Sample

1. Use the objects to create an array with 2 rows.

a.	Array with 2 rows	Redraw your picture with 1 *less* star.
	 12	 11
	There are an even/odd (circle one) number of stars.	There are an even/odd (circle one) number of stars.